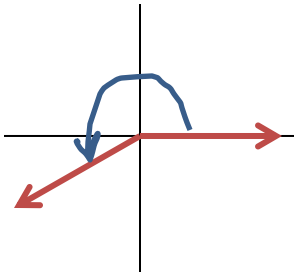
Reference Angles

Ex1 : Find sin 215°.



How do you draw a triangle with a 215° angle in it?

In order to solve trig problems with angles bigger,

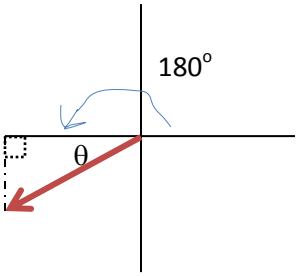
We use a reference angle.

To make the triangle → ALWAYS draw a side from the end of the terminal side either DOWN or UP to the x-axis.

The reference angle is the acute angle in this triangle.

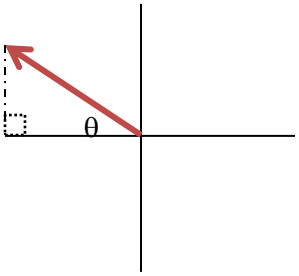
To find the reference angle for:

a 215° angle,



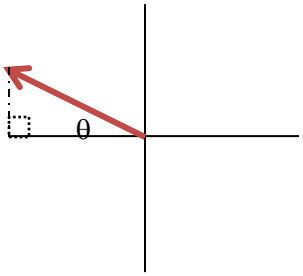
215 – 180 = 35
Reference angle = 35°

a 500° angle,



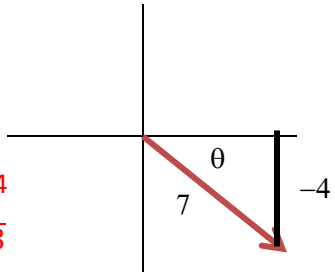
500 – 360 = 140
180 – 140 – 40
Reference angle = 40°

a –200° angle



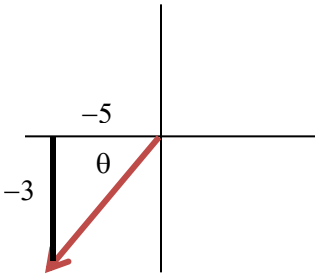
–200 +180 = 20
Reference angle = 20°

Ex: If $sin\theta = -\frac{4}{7}$, and the terminal side of θ lies in quadrant IV, find $cos\theta$.



1. Draw a ray in quadrant IV
2. Draw a line segment UP to the x-axis
3. Label the reference angle θ
4. $sin\theta = -\frac{4}{7} = \frac{opp}{hyp}$. The hypotenuse is ALWAYS positive, so hyp = 7 and opp = –4
5. Find the adjacent side using Pythagorean Thm→ $x^2 + (-4)^2 = 7^2$ → $x = \pm\sqrt{33}$
6. Looking at the triangle, the adjacent side is in the positive direction, so $x = \sqrt{33}$
7. $cos\theta = \frac{adj}{hyp} = \frac{\sqrt{33}}{7}$

Ex: If $tan\theta = \frac{3}{5}$, and the terminal side of θ lies in quadrant III, find $sin\theta$.



1. Draw a ray in quadrant III
2. Draw a line segment UP to the x-axis
3. Label the reference angle θ
4. $tan\theta = \frac{3}{5} = \frac{opp}{adj}$. Tan is positive, so the numerator and denominator must have the same sign. The opposite side AND the adjacent side are pointing in the negative direction, so they BOTH must be negative.
5. Find the hypotnuse using Pythagorean Thm→ $(-5)^2 + (-3)^2 = x^2$ → $x = \pm\sqrt{34}$
6. The hypotenuse is ALWAYS positive, so $x = \sqrt{34}$
7. $sin\theta = \frac{opp}{hyp} = \frac{-3}{\sqrt{34}} = \frac{-3\sqrt{34}}{34}$